**QUANTUM FIELD THEORY I**
**written test**

June 28, 2019

*Two hours. No books or notes allowed.*

Consider a theory of a Dirac fermion $f$ with field $\psi$ and a real scalar field $s$ with field $\phi$, with Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - M^2\phi^2\right) + \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + g\bar{\psi}\psi\phi. \tag{1}$$

(1) Determine the energy-momentum tensor for this theory.
(2) Determine the internal symmetry or symmetries for this theory (namely, those wich leave the space-time coordinates unchanged) and the corresponding Noether current.
(3) Express the Noether current(s) determined in question 2 in terms of creation and annihilation operators.
(4) Write down the Feynman rules for this theory and determine whether it is renormalizable or not.
(5) Write down at leading nonvanishing order the Feynman diagrams and compute the square modulus of the unpolarized amplitude for the process $fs \to fs$, expressing the result in terms of Mandelstam invariants.
(6) Discuss whether when $m = M = 0$ the *internal* symmetries of the theory are greater than for a generic choice of mass values.
(7) Compute the trace of the energy momentum tensor of the (classical) theory and show (using the classical equations of motion) that the trace vanishes for a particular choice of the parameters $g$, $M$, $m$.
*Hint:* Integrate by parts and use the classical equations of motion.