QUANTUM FIELD THEORY I
written test

January 16, 2020

Two hours. No books or notes allowed.

Consider a theory with Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) + \bar{\psi}\left(i\partial\!\!\!/ - m_f\right)\psi - g'\bar{\psi}\gamma^\mu\psi B_\mu + \frac{g}{4}\phi F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where $F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field, ϕ is a real scalar field, ψ is a Dirac fermion and B_μ is a vector field, assumed external, i.e., such that its free Lagrangian is not part of the Lagrangian Eq. (1).

(1) Write down the Feynman rules for this theory and determine whether it is renormalizable.

(2) Determine the energy-momentum tensor and the Hamiltonian density for this theory.

(3) Determine to lowest nontrivial perturbative order the unpolarized squared amplitude for the process

$$\gamma(p_1) + \gamma(p_2) \to \phi(k), \quad (2)$$

i.e., the production of a scalar field in the annihilation of a photon-photon pair. Express the result in terms of Mandelstam invariants. Perform the sum over the polarization of the photon using $\sum_s \epsilon_s^\mu(p)\epsilon_s^{*\nu}(p) = -g^{\mu\nu}$.

(4) Determine the cross-section for the process given at point 3, in the center-of-mass reference frame of the two colliding photons.

(5) Consider now the process

$$f(p_1) + \bar{f}(p_2) \to B(k), \quad (3)$$

where f is the Dirac fermion with field ψ in the Lagrangian Eq. (1), i.e. the production of a vector field in the annihilation of a fermion-antifermion pair. Assume that the vector field B is massive, with mass m_B. Determine the amplitude and the cross-secttion for this process to lowest perturbative order.

(6) Compare the results found at points 4 and 5, and discuss in each case the dependence of the cross-section found on the kinematic variables. Specifically, discuss on how many independent variables the cross-section depends upon, and why.

(7) Consider again the process at point (5). Assume now that each of the two incoming fermions is part of a flux of incoming fermions (such as e.g. in a particle accelerator). The fermions in the flux have a distribution of probability of momenta, such that each of them can have momentum $p_i = x_i p^{\max}$, with $0 \le x_i \le 1$, with probability $p(x_i)$, assumed to be the same for both fermions. Determine the total cross-section integrating over the momentum probability distribution.