FISICA TEORICA I

january 23, 2024

Closed-book, 2 hours time

Consider a theory with a real massive scalar field ϕ and a massless Dirac fermion f with field ψ. The Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m\phi^2\right) + \bar{\psi}\left(i\partial\!\!\!/ + ig(a + b\gamma_5)\phi\right)\psi. \tag{1}$$

(1) Determine the energy-momentum tensor and the energy density for this theory.

(2) Determine the internal symmetries of the theory, the associate classical Noether currents and in the quantized case express the charge operators for these currents in terms of field creation and annihilation operators.

(3) Write down the Feynman rules for this theory.

(4) Determine the first-order amplitude $M(p_1, p_2; p_3, p_4)$ for the process $f\bar{f} \to f\bar{f}$ (fermion-antifermion elastic scattering), where p_1, p_2 and p_3, p_4 are respectively the momenta of the incoming and outgoing fermion and antifermion. If there is more than one Feynman diagram, discuss the relative sign.

(5) Determine the unpolarized square amplitude $A(p_1, p_2; p_3, p_4)$ for the given process, writing the answer in terms of traces over gamma matrices, and prove that all terms with an odd number of γ_5 matrices either vanish or cancel. Prove that the remaining traces are all proportional to $(a^2 - b^2)^2$.

(6) Compute the traces over gamma matrices and express the amplitude $A(p_1, p_2; p_3, p_4)$ in terms of scalar products of the four-vectors p_i.

(7) Express $A(p_1, p_2; p_3, p_4)$ in terms of Mandelstam invariants.

(8) Determine the Mandelstam invariants when $p_3 = p_1$; $p_4 = p_2$, or $p_3 = p_2$; $p_4 = p_1$ and compute in terms of them the forward-backward asymmetry

$$A_{FB}(p_1, p_2) = A(p_1, p_2; p_1, p_2) - A(p_1, p_2; p_2, p_1).$$