QUANTUM FIELD THEORY I
written test

June 30, 2017

Two hours. No books or notes allowed.

Consider a theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + g\bar{\psi}\gamma^\mu\gamma_5\psi\partial_\mu\phi \tag{1}$$

where ϕ is a real scalar and ψ a Dirac fermion field.

(1) Write down the Feynman rules for this theory and determine the dimensionality of the coupling g.

(2) Determine to first order in g the matrix element for the process

$$f\left(p_1\right) + \bar{f}\left(p_2\right) \to \phi\left(p_3\right) + \phi\left(p_4\right), \tag{2}$$

namely fermion-antifermion annihilation in a pair of scalars. Express the result in terms of Mandelstam invariants.

Hints:

- In order to perform the calculation, prove the identity

$$\bar{v}\left(p_2\right)\left[p\!\!\!/_3 + p\!\!\!/_4\right]u\left(p_1\right) = 0 \tag{3}$$

where u and v are standard positive- and negative-energy solutions of Dirac equation.

- Recall the identity

$$p\!\!\!/q\!\!\!/ = 2\left(p\cdot q\right) - q\!\!\!/p\!\!\!/. \tag{4}$$

- The final result for the amplitude is

$$i\mathcal{M} = -2mig^2\bar{v}\left(p_2\right)\left[\frac{p\!\!\!/_4 p\!\!\!/_3}{t-m^2} + \frac{p\!\!\!/_3 p\!\!\!/_4}{u-m^2}\right]u\left(p_1\right). \tag{5}$$

(3) Determine the unpolarized square amplitude for the given process using the result of the previous question; express the result in terms of Mandelstam invariants.
Hint: Reorder the traces using Eq. (4) and take advantage of the identities

$$p\!\!\!/_3 p\!\!\!/_3 = p_3^2 = 0, \tag{6}$$

$$p\!\!\!/_4 p\!\!\!/_4 = p_4^2 = 0. \tag{7}$$

(4) Determine the phase-space and the unpolarized differential cross-section in the center-of-mass frame for the given process, using the result of the previous question.

(5) Discuss whether the given theory is renormalizable. Write down the most general Lorentz-invariant interaction built out of the fields of the given theory, and whose couplings have all the same dimension as g.

(6) Discuss the $m \to 0$ limit of the given theory. In particular, determine the internal symmetries of the theory, comparing the two cases $m \neq 0$ and $m = 0$, and determine the conserved Noether currents in either case. How does the cross-section of point (4) behave in this limit, and why?