FISICA TEORICA I

june 28, 2023

Closed-book, 2 hours time

Consider a theory with a real scalar field ϕ and a Dirac fermion f with field ψ. The Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2\right) + \bar{\psi}\left(i\partial\!\!\!/ - m + ig\phi\right)\psi. \tag{1}$$

(1) Determine the energy-momentum tensor and the Hamiltonian density for this theory.

(2) List the internal symmetries (namely, those that leave space-time coordinates unaffected), determine the corresponding classical Noether currents, and in the quantized case express the charge operators for these currents in terms of field creation and annihilation operators.

(3) Write down the Feynman rules for this theory.

(4) Draw the Feynman diagrams for the process $f\phi \to f\phi$ ("Compton" scattering) and write down the corresponding amplitudes.

(5) Determine the square modulus of the unpolarized amplitude for this process in terms of the momenta of incoming and/or outgoing particles, in the limit in which the fermion mass is negligible, $m \to 0$.

(6) Express the result found at the previous point in terms of Mandelstam invariants.

(7) Consider now the case in which the Lagrangian is

$$\mathcal{L}' = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2\right) + \bar{\psi}\left(i\partial\!\!\!/ - m + ig\gamma_5\phi\right)\psi \tag{2}$$

(pseudoscalar interaction). Repeat points (3-7) in this case, verify whether the result changes or not, and discuss why.

(8) Determine again the square modulus of the unpolarized amplitude as in point (5), but now in the opposite limit, i.e. in the case in which the fermion mass goes to infinity, $m \to \infty$. Discuss the result.