QUANTUM FIELD THEORY I
written test

July 20, 2017

Two hours. No books or notes allowed.

Consider a theory with Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) + \frac{g}{4}\phi F_{\mu\nu}F^{\mu\nu} \tag{1}$$

where $F^{\mu\nu}$ is the electromagnetic field and ϕ is a real scalar field (to be called "Higgs", henceforth).

(1) Write down the Feynman rules for this theory and determine the dimensionality of the coupling g.

(2) Determine to lowest nontrivial perturbative order the unpolarized squared amplitude for the process

$$\gamma\,(p_1) + \gamma\,(p_2) \to \phi\,(k) \tag{2}$$

i.e. Higgs production in photon fusion. Express the result in terms of Mandelstam invariants.
Hint: the sum over polarizations is given by $\sum_s \epsilon^\mu_s(p)\epsilon^{*\nu}_s(p) = -g^{\mu\nu}$.

(3) Consider now to lowest nontrivial perturbative order in the given theory the process

$$\gamma\,(p_1) + \gamma\,(p_2) \to \gamma\,(k_1) + \gamma\,(k_2) \tag{2}$$

i.e. photon-photon elastic scattering. Draw all the Feynman diagrams and write down the amplitude for this process in terms of the four momenta p_1, p_2, k_1, k_2.

(4) Determine, for the process at point (3), the square modulus of the contribution to the amplitude from the s-channel diagram only, neglecting the contributions from all other diagrams. Express the result in terms of Mandelstam invariants.

(5) Determine, for the process at point (2), the cross-section in the center-of-mass frame.

(6) Also in the center-of-mass frame, determine the flux factor and the phase space for the process at point (3), and write down the cross-section, expressed as a sum of amplitudes without providing the explicit expression for the amplitudes. Only the square modulus of the s-channel contribution was determined at point (4): discuss how you expect the contributions to the amplitue from other diagrams to look like.

(7) Prove the relation

$$\mathrm{Im} M_2 = \frac{\Phi}{2}\sigma_1, \tag{3}$$

where

$$M_2 = \frac{1}{4}\sum_{s_1 s_2} \epsilon^\mu_{s_1}(p_1)\epsilon^\nu_{s_2}(p_2) M_{\mu\nu,\rho\sigma}(p_1,p_2;p_1,p_2)\epsilon^{*\rho}_{s_1}(p_1)\epsilon^{*\sigma}_{s_2}(p_2) \tag{4}$$

and $M_{\mu\nu,\rho\sigma}(p_1,p_2;k_1,k_2)$ is the s-channel amplitude computed at point (4); and σ_1 and Φ are the cross-section and flux factor computed at point (5). Explain the origin of this relation in terms of Feynman diagrams.
Hint: Recall the identity

$$\lim_{\epsilon\to 0}\frac{1}{x+i\epsilon} = \frac{1}{x} - i\pi\delta(x). \tag{5}$$