QUANTUM FIELD THEORY I
written test

July 23, 2019

Two hours. No books or notes allowed.

Consider scalar electrodynamics, whose Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi, \tag{1}$$

with ϕ a complex scalar field, $F^{\mu\nu}$ is the Maxwell field strengh tensor and the covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu + ieA_\mu. \tag{2}$$

(1) Determine the Hamiltonian density $\mathcal{H} = T^{00}$ for this theory, where $T^{\mu\nu}$ is the energy-momentum tensor, not including the contribution from the Maxwell Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

(2) Determine the internal symmetry or symmetries for this theory (namely, those wich leave the space-time coordinates unchanged) and the corresponding (classical) Noether current.

(3) Write down the Feynman rules for this theory and determine whether it is renormalizable or not.

(4) Write down at leading nonvanishing order the Feynman diagrams and compute the amplitude for the process $\gamma\phi \to \gamma\phi$

(5) Prove by explicit computation that the amplitude computed at the previous point vanishes if one of the polarization vectors for the external photons is replaced by the momentum carried by that photon (it is sufficient to perform the check for one of the two photons).

(6) Compute the square modulus of the amplitude determined at point (4) for an unpolarized process. The sum over polarizations can be performed using $\sum_\lambda \epsilon^*_{\mu,\lambda}(p_4)\epsilon_{\rho,\lambda}(p_4) = -g_{\mu\rho}$.

(7) Determine the canonical commutation relations for this theory, and use the result to compute the commutator of the Noether charge(s) determined in question (2) with all the fields of the theory. What is the meaning of the result?