

ESAME SCRITTO DI FISICA TEORICA I

19 luglio 2023

Tempo massimo 2 ore. Non sono ammessi libri o appunti

Consider quantum electrodynamics with two fermion fields with the same electromagnetic coupling but unequal masses. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_1 (i\not{D} - m_1) \psi_1 + \bar{\psi}_2 (i\not{D} - m_2) \psi_2, \quad (1)$$

where $D_\mu = \partial_\mu + ieA_\mu$.

- (1) Determine the energy-momentum tensor and the Hamiltonian density for this theory. Simplify the result by choosing the Coulomb gauge $A^0 = 0$.
- (2) List the internal symmetries (namely, those that leave space-time coordinates unaffected), determine the corresponding classical Noether currents, and in the quantized case express the charge operators for these currents in terms of field creation and annihilation operators. A full derivation of the expression of the operators is not requested but the main steps should be briefly outlined.
- (3) Write down the Feynman rules for this theory.
- (4) Draw the Feynman diagrams for the process $f\bar{f} \rightarrow \gamma\gamma$, i.e. for the production of a pair of photons in fermion-antifermion pair annihilation, where “fermion” denotes any of the two fermion fields.
- (5) Determine the square modulus of the unpolarized amplitude for this process in terms of the momenta of incoming and/or outgoing particles, in the limit in which the fermion mass is negligible, $m \rightarrow 0$, and discuss whether the result depends on the nature of the incoming fermion. The sum over the polarizations of the outgoing photons can be performed using $\sum_\lambda \epsilon_\mu^*(p, \lambda)\epsilon_\nu^*(p, \lambda) = -g_{\mu\nu}$
- (6) Express the result found at the previous point in terms of Mandelstam invariants.
- (7) Determine now the square modulus of the unpolarized amplitude for the process $\gamma\gamma \rightarrow f\bar{f}$, assuming that the nature of the outgoing fermion cannot be determined.
- (8) Assume now that the masses of the two fermions vanish identically. Determine all the internal symmetries in this case, list all the conserved charges and write all the commutation relations between them.