**QUANTUM FIELD THEORY I**
**written test**

September 27, 2017

*Two hours. No books or notes allowed.*

Consider a theory with two Dirac fermions $e$ e $\nu$ with fields $\psi_e$ and $\psi_\nu$ respectively, coupled to an electromagnetic field $A^\mu$, with Lagrangian

$$\mathcal{L} = \bar{\psi}_e \left(i\partial\!\!\!/ - m_e\right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + g\bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_\nu A_\mu + g\bar{\psi}_\nu \gamma^\mu (1-\gamma_5)\psi_e A_\mu, \quad (1)$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad (2)$$

and $g$ is a real positive constant.

(1) Write down the Feynman rules for this theory.

(2) Draw the Feynman diagrams for the processes $e\bar{\nu} \to e\bar{\nu}$ and $e\nu \to e\nu$ at the lowest nontrivial order in perurbation theory and write down the corresponding amplitudes.

(3) Determine the square modulus of the unpolarized amplitude for the two given processes in terms of Mandelstam invariants keeping the full mass dependence.
*Hint:* recall the identity $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\alpha\beta} = -2\left(g^\rho{}_\alpha g^\sigma{}_\beta - g^\rho{}_\beta g^\sigma{}_\alpha\right)$.

(4) Determine the phase space and flux factor for the given processes in the lab frame.
*Hint:* Exploit energy conservation to eliminate the dependence of the phase space on $\cos\theta$ and write the result in terms of the energies $E_\nu$ and $E_e$ of the incoming $\nu$ and outgoing $e$.

(5) Define the variable $y = \frac{E_e}{E_\nu}$ and determine explicitly the differential cross-section $\frac{d\sigma}{dy}$ as a function of $E_\nu$ and $y$.

(6) Prove that in the limit $m_e \to 0$ the Lagrangian Eq. (1) has an internal U(1)×U(1) symmetry.
*Hint:* Define the doublet $\psi = \begin{pmatrix} \psi_e \\ \psi_\nu \end{pmatrix}$ and rewrite the Lagrangian in terms of $\psi$ using Pauli matrices.