QUANTUM FIELD THEORY I
written test

September 20, 2019

Two hours. No books or notes allowed.

Consider a theory with two massless Dirac fermions f_1 and f_2 with fields ψ_1 and ψ_2 respectively, with Lagrangian

$$\mathcal{L} = \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + G\bar{\psi}_1\gamma^\mu\psi_2\bar{\psi}_2\gamma^\mu\psi_1 \tag{1}$$

where G is a real constant.

(1) Write down the Feynman rules for this theory and determine whether it is renormalizable or not.

(2) Determine the internal symmetry or symmetries and conservation laws for this theory. At the classical level, write down the conserved Noether currents, and at the quantum level, write down the conserved operator charges in terms of creation and annihilation operators.

(3) Draw the Feynman diagrams for the processes $f_1\bar{f}_2 \to f_1\bar{f}_2$ and $f_1 f_2 \to f_1 f_2$ (where f is the fermion and $\bar{f}$ the antifermion) and write down the corresponding amplitudes.

(4) Compute the square modulus of the unpolarized amplitudes for the two processes in the previous question. Discuss the relation between them.

(5) Add to the Lagrangian an extra contribution of the form

$$\Delta\mathcal{L} = G T^\mu T^{\dagger\mu}, \tag{2}$$

where

$$T^\mu \equiv \lambda_1\bar{\psi}_1\gamma^\mu\psi_1 + \lambda_2\bar{\psi}_2\gamma^\mu\psi_2, \tag{3}$$

and determine the values of the parameters λ_i such that the Lagrangian can be written as

$$\mathcal{L} = \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + \frac{G}{4}\sum_{i,a,b,a',b'} \left(\bar{\psi}_a\gamma^\mu\sigma^i_{ab}\psi_b\right)\left(\bar{\psi}_{a'}\gamma^\mu\sigma^i_{a'b'}\psi_{b'}\right), \tag{4}$$

where σ^i_{ab} are components of the i-th Pauli matrix (so a, b, a', $b' = 1, 2$ while $i = 1, 2, 3$).

(6) Determine the internal symmetries and associated Noether currents for the Lagrangian Eq. (4).

(7) Determine the equal-time commutators between the time components ($\mu = 0$) of any two of the Noether currents determined at point (6).
Hint: use the canonical commutation relations.

(8) Write down the Feynman rules for the Lagrangian Eq. (4).