ESAME SCRITTO DI FISICA TEORICA I

13 settembre 2023

Tempo massimo 2 ore. Non sono ammessi libri o appunti

Consider a theory with three real scalar fields ϕ_1, ϕ_2, ϕ_3 with unequal masses, $m_3 > m_2 > m_1$. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\left[\left(\partial_\mu\phi_1\partial^\mu\phi_1 - m_1^2\phi_1^2\right) + \left(\partial_\mu\phi_2\partial^\mu\phi_2 - m_2^2\phi_2^2\right) + \left(\partial_\mu\phi_3\partial^\mu\phi_3 - m_3^2\phi_3^2\right)\right] + g\phi_1\phi_2\phi_3 \tag{1}$$

(1) Determine the energy-momentum tensor and the Hamiltonian density for this theory.
(2) Discuss whether the theory is renormalizable or not.
(3) Write down the Feynman rules for this theory.
(4) Consider the set of processes $\phi_1\phi_1 \to XY$, where X and Y can be any of the fields ϕ_1, ϕ_2, ϕ_3. Determine which of these processes, for all possible choices of X and Y, can have a nonvanishing amplitude at tree level (i.e. with no loops).
(5) Draw the Feynman diagrams for all the processes with nonvanishing amplitude determined at the previous point.
(6) Determine the amplitudes corresponding to the diagrams at the previous point in terms of scalar products beween the momenta of the incoming and outgoing particles.
(7) Express the result of the previous point in terms of Mandelstam invariants.
(8) Consider now the set of processes $\phi_1\phi_2 \to XY$. Determine again which of these have nonvanishing amplitude, and determine these nonvanishing amplitudes in terms of momenta and in terms of Mandelstam invariants.
(9) Determine the relation between the amplitudes at point (6) and those at point (8) expressed in terms of momenta of incoming or outgoing particles.
(10) Determine the physical region, i.e. the range of values of the momenta of the incoming particles for which the amplitudes of point (8) are nonzero.
(11) Determine whether there is a region in which the amplitudes of point (6) and those of point (8) can have a nonvanishing imaginary part.