Scritto di fisica teorica I

19-1-2022

Traccia di soluzione

1)

$$T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi + i \bar{\psi} \gamma^\mu \partial^\nu \psi - g^{\mu\nu} \mathcal{L} \,. \qquad (1)$$

Campi canonici

$$\pi_\varphi = \dot{\varphi} \,; \qquad (2)$$

$$\pi_\psi = i \bar{\psi} \gamma^0 = i \psi^+ . \qquad (3)$$

$$\mathcal{H} = T^{00} = T_\varphi + T_\psi + V \,. \qquad (4)$$

$$T_\varphi = \frac{1}{2} \dot{\varphi}^2 = \frac{1}{2} \pi^2 \,. \qquad (5)$$

$$T_\psi = i \psi^+ \dot{\psi} = \pi \dot{\psi} \qquad (6)$$

$$V = \frac{1}{2} M^2 \varphi^2 + \frac{\lambda}{4!} \varphi^4 + \frac{1}{2} \bar{\nabla} \varphi \cdot \bar{\nabla} \varphi + \bar{\psi}\, i \bar{\gamma} \cdot \bar{\nabla} \psi + m \bar{\psi} \psi - i g \bar{\psi} \gamma_5 \psi \phi \qquad (7)$$

so

$$\mathcal{H} = \frac{1}{2} \left(\pi^2 + \bar{\nabla} \varphi \cdot \bar{\nabla} \varphi + M^2 \varphi^2 \right) + i \bar{\psi} \bar{\gamma} \cdot \bar{\nabla} \psi + m \bar{\psi} \psi + \frac{\lambda}{4!} \phi^4 - i g \bar{\psi} \gamma_5 \psi \phi \qquad (8)$$

2) L'unica simmetria interna della teoria è U(1) di fase per il campo fermionico

$$\psi \to \psi' = e^{i\theta}\,\psi \qquad (9)$$

corrispondente alla conservazione della corrente fermionica

$$j^\mu = \bar\psi\,\gamma^\mu\,\psi \qquad (10)$$

e a livello quantistico del numero fermionico

$$N_\psi = \sum_{r,s}\int\frac{d^3p}{(2\pi)^3}\left(b^{\dagger(s)}_{\bar p}\,b^{(s)}_{\bar p} - d^{\dagger(r)}_{\bar p}\,d^{(r)}_{\bar p}\right) \qquad (11)$$

avendo posto

$$\psi(x) = \int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\,\sum_{r,s}\left(b^{(s)}_{\bar p}\,u^{(s)}(\bar p)\,e^{-ipx} + d^{\dagger r}_{\bar p}\,v^{(r)}(\bar p)\,e^{ipx}\right) \qquad (12)$$

3) <u>Propagatori</u>

scalare $\;-\,-\to_{p}-\,-\qquad \dfrac{i}{p^2 - M^2 + i\varepsilon}$ (13)

fermione $\;\xrightarrow[k]{}\qquad \dfrac{i(\not k + m)}{k^2 - m^2 + i\varepsilon}$ (14)

<u>Linee esterne</u>

$\underset{p}{-\,-\,-\bullet} = \bullet\,-\,-\,-\qquad 1$ (15)

$\underset{k}{\longrightarrow\bullet}\quad u^{(s)}(k)\qquad\qquad \bullet\!\longrightarrow\quad \bar u^{(s)}(k)$ (16)

$\longleftarrow\bullet\quad \bar v^{(r)}(k)\qquad\qquad \bullet\!\longleftarrow\quad v^{(r)}(k)$ (17)

Vertici

 $-i\lambda$ (18)

 $-g\gamma_5$ (19)

4) 4 dimensioni (20)

$[\varphi] = [M]$ (21)

$[\psi] = [M]^{3/2}$ (22)

Ne segue

$[\varphi^4] = [M]^4 \Rightarrow [\lambda] = [M]^0$ (8)

$[\bar{\psi}\gamma_5\psi\phi] = [M]^4 \Rightarrow [g] = [M]^0$

Costanti d'accoppiamento adimensionali $\rightarrow$ teoria rinormalizzabile

6 dimensioni

$[\varphi] = [M]^2$ $\left(\text{Check}: [\partial_\mu\varphi\partial_\mu\varphi] = [M]^6\right)$ (24)

$[\psi] = [M]^{5/2}$ $\left(\text{Check } [\bar{\psi}\partial\psi] = = [M]^6\right)$ (25)

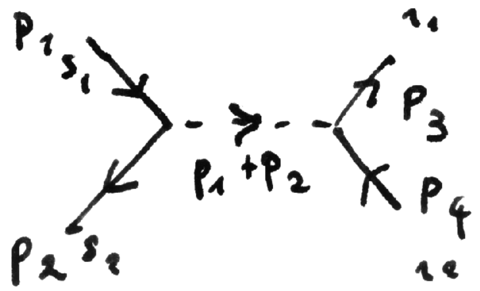
Ne segue (26)

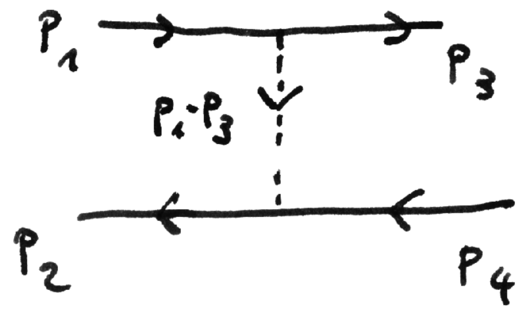
$[\varphi^4] = [M]^8 \Rightarrow [\lambda] = [M]^{-2}$

$[\bar{\psi}\gamma_5\psi\phi] = [M]^7 \Rightarrow [g] = [M]^{-1}$ (27)

Entrambi gli accoppiamenti sono non-rinormalizzabili

5)





$$i\mathcal{M}_s = \bar{u}^{r_1}(p_3)\, g\gamma_5\, v^{r_2}(p_4)\, \frac{i}{s-M^2+i\varepsilon}\, \bar{v}^{s_2}(p_2)\, g\gamma_5\, u^{(s_1)}(p_1) \qquad (28)$$

$$i\mathcal{M}_t = \bar{u}^{r_1}(p_3)\, g\gamma_5\, u^{s_1}(p_1)\, \frac{i}{t-M^2+i\varepsilon}\, \bar{v}^{s_2}(p_2)\, g\gamma_5\, v^{(r_2)}(p_4) \qquad (29)$$

$$i\mathcal{M} = i\left(\mathcal{M}_s - \mathcal{M}_t\right) \qquad (30)$$

(notare il segno − dovuto alla permutazione nell'ordine dei fermioni)

6) $$A = \frac{1}{4}\sum_{rs} \left|\mathcal{M}_s - \mathcal{M}_t\right|^2 =$$

$$= \frac{g^4}{4}\Bigg[\frac{\mathrm{Tr}\,\gamma_5(\not p_1+m)\gamma_5(\not p_2-m)\ \mathrm{Tr}\,\gamma_5(\not p_4-m)\gamma_5(\not p_3+m)}{(s-M^2)^2} +$$

$$+ \frac{\mathrm{Tr}\,\gamma_5(\not p_4-m)\gamma_5(\not p_2-m)\ \mathrm{Tr}\,\gamma_5(\not p_1+m)\gamma_5(\not p_3+m)}{(t-M^2)^2}$$

$$- \frac{\mathrm{Tr}\,\gamma_5(\not p_1+m)\gamma_5(\not p_3+m)\gamma_5(\not p_4-m)\gamma_5(\not p_2-m)}{(s-M^2)(t-M^2)}$$

$$- \frac{\mathrm{Tr}\,\gamma_5(\not p_4-m)\gamma_5(\not p_3+m)\gamma_5(\not p_1+m)\gamma_5(\not p_2-m)}{(s-M^2)(t-M^2)}\Bigg] \qquad (31)$$

7) $$A = g^4 \left(4\frac{p_1\cdot p_2 \; p_3\cdot p_4}{(s-M^2)^2} + 4\frac{p_1\cdot p_3 \; p_2\cdot p_4}{(t-M^2)^2} + \right.$$

$$- \frac{1}{(s-M^2)(t-M^2)} \Big[\left(p_1\cdot p_3 \; p_4\cdot p_2 - p_1\cdot p_4 \; p_3 p_2 + p_1\cdot p_2 \; p_3 p_4 \right)$$
$$\left. + \left(p_4\cdot p_3 \; p_1\cdot p_2 - p_4\cdot p_1 \; p_3 p_2 + p_4\cdot p_2 \; p_1\cdot p_3 \right) \Big] \right) \qquad (32)$$

$$\frac{s}{2} = p_1\cdot p_2 = p_3\cdot p_4 \qquad (33)$$

$$\frac{t}{2} = p_1\cdot p_3 = p_2\cdot p_4 \qquad (34)$$

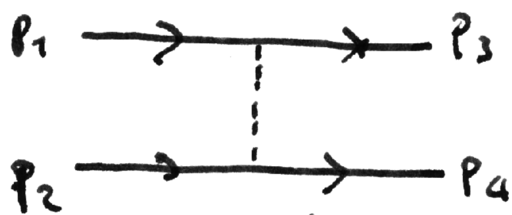
$$\frac{u}{2} = p_1\cdot p_4 = p_2\cdot p_3 \qquad (35)$$

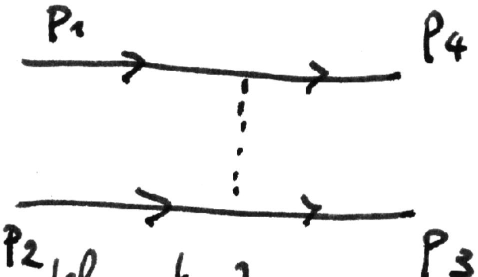
$$A = \frac{g^4}{4} \left(\frac{4s^2}{(s-M^2)^2} + \frac{4t^2}{(t-M^2)^2} - \frac{2}{(s-M^2)(t-M^2)} (t^2 - u^2 + s^2) \right)$$

$$= g^4 \left(\frac{s^2}{(s-M^2)^2} + \frac{t^2}{(t^2-M^2)^2} - \frac{st}{2(s-M^2)(t-M^2)} \right) \qquad (36)$$

avendo usato $(s+t+u)=0 \Rightarrow s+t=-u$

8) Nel caso $ff \to ff$ i diagrammi sono





Ne segue che l'ampiezza è la stessa del punto 7, ma con $s \Leftrightarrow u$