**QUANTUM FIELD THEORY II**
**PROBLEMS**
1st semester 2025-2026

*I: The optical theorem*

(1) Consider the one-loop $2 \to 2$ $M(p_1, p_2; k_1, k_2)$ amplitude in $\phi^4$ theory, given by

$$iM(p_1, p_2; k_1, k_2) = -i\lambda \left[1 - i\lambda \left(V(s) + V(t) + V(u)\right)\right], \tag{1}$$

with, in the $\overline{\text{MS}}$ scheme,

$$-iV(p^2) = \frac{\lambda}{32\pi^2} \int_0^1 dx \ln \frac{m^2 - p^2 x(1-x)}{\mu^2}. \tag{2}$$

Determine explicitly the imaginary part of the amnplitude and check that it satisfies the optical theorem.

(2) Using canonical commutation relations, show that the momentum operator $P^\mu = \int d^3x T^{0\mu}$, where $T^{\mu\nu}$ is the energy momentum tensor, generates translation of the fields upon commutation:

$$[P^\mu, \phi(x)] = -i\partial^\mu \phi(x). \tag{3}$$